FORMULARI

28-7-2010

Tema 1. Fonaments físico-matemàtics de la mecànica

$$\vec{\nabla} U = \left(\frac{\partial U}{\partial x}, \frac{\partial U}{\partial y}, \frac{\partial U}{\partial z} \right) \qquad dU = \vec{\nabla} U \cdot d\vec{r} + \frac{\partial U}{\partial t} dt$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \left(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt} \right) \qquad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2} = \left(\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}, \frac{d^2z}{dt^2} \right) \qquad \vec{a} = \frac{dv}{dt}\hat{v} + \frac{v^2}{R}\hat{n}$$

Tema 2. Mecànica d'una partícula

$$\vec{F} = m\vec{a} \qquad \vec{p} = m\vec{v} \qquad I = \int_{t_1}^{t_2} \vec{F}\, dt \qquad \frac{d\vec{p}}{dt} = \vec{F} \qquad I = \Delta\vec{p}$$

$$\vec{M}_{(P)} = \vec{r}_{(P)} \times \vec{F} \qquad \vec{L}_{(P)} = \vec{r}_{(P)} \times \vec{p} \qquad \vec{Y}_{(P)} = \int_{t_1}^{t_2} \vec{M}_{(P)}\, dt \qquad \frac{d\vec{L}_{(P)}}{dt} = \vec{M}_{(P)} \qquad \vec{Y}_{(P)} = \Delta\vec{L}_{(P)}$$

$$W = \int_{C:P_1}^{P_2} \vec{F} \cdot d\vec{r} \qquad E_c = \frac{1}{2} m v^2 \qquad W = \Delta E_c$$

força conservativa: $dW = \vec{F} \cdot d\vec{r} = -dU$

$$E = E_c + U \qquad P = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$$

Tema 3. Mecànica d'N partícules

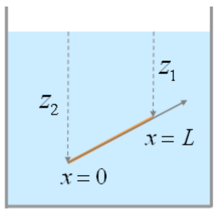
$$\vec{r}_{CM} = \frac{1}{m} \sum_{i=1}^{N} m_i \vec{r}_i \qquad \vec{r}_{CM} = \frac{1}{m} \int_{Cos} \vec{r}\, dm \qquad e = \left| \frac{\hat{n} \cdot \left(\vec{v}_{1(fi)} - \vec{v}_{2(fi)} \right)}{\hat{n} \cdot \left(\vec{v}_{1(ini)} - \vec{v}_{2(ini)} \right)} \right| \qquad \sum_{i=1}^{N} \left(\vec{F}_i - m_i \vec{a}_i \right) \cdot \delta\vec{r}_i = 0$$

sòlid rígid: $d\vec{r}_i = d\vec{r}_C + d\vec{\varphi} \times \vec{r}_{(C)i}$ $\qquad \vec{v}_i = \vec{v}_C + \vec{\omega} \times \vec{r}_{(C)i}$

Tema 4. Estàtica dels sòlids rígids

$$\vec{F} = 0 \qquad \vec{M}_{(O)} = 0 \qquad \vec{M}_{(O')} = \vec{M}_{(O)} - \overrightarrow{OO'} \times \sum_{i=1}^{N} \vec{F}_i$$

$$dp = \rho g dz \qquad E = P_{fluid} = \rho g V_{fluid} \qquad F = \rho g \frac{1}{2} L (z_2 + z_1) a \qquad x_F = \frac{L(z_2 + 2z_1)}{3(z_2 + z_1)}$$

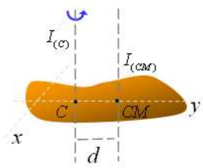


$$\sum_a \vec{F}_a \cdot \delta\vec{r}_a = 0 \qquad \frac{\partial U}{\partial \lambda_i} = 0 \quad (i = 1, \ldots, L)$$

Tema 5. Dinàmica del sòlid rígid en el pla

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt} \qquad \vec{v}_i - \vec{v}_j = \vec{\omega} \times \vec{r}_{ji} \text{ on } \vec{r}_{ji} = \vec{r}_i - \vec{r}_j \qquad \vec{r}_{(C)EIR} = \lambda\vec{\omega} + \left(\frac{\vec{\omega} \times \vec{v}_C}{\omega^2} \right)$$

$$\vec{v}_{EIR} = \frac{\vec{\omega} \cdot \vec{v}_C}{\omega^2} \vec{\omega}$$



$$I_{(C)} = \sum_{i=1}^{N} m_i r_{(|C)i}^2 \qquad L_{(C)} = I_{(C)} \omega \qquad M_{(C)} = I_{(C)} \alpha \qquad I_{(C)} = I_{(CM)} + md^2$$

$$E_C = \frac{1}{2} m v_{CM}^2 + \frac{1}{2} I_{(CM)} \omega^2$$

Tema 6. Petites oscil·lacions

$\ddot{x}+\omega_0^2 x=0$ $\quad x(t)=A\sin\left(\omega_0 t+\varphi_0\right)$ $\quad E=\frac{1}{2}m\omega_0^2A^2=\frac{1}{2}kA^2$

$\ddot{x}+2\gamma\dot{x}+\omega_0^2 x=0$

$\omega_0>\gamma\to x(t)=Ae^{-\gamma t}\sin\left(\omega t+\varphi_0\right)\to\omega^2=\omega_0^2-\gamma^2$

$\omega_0<\gamma\to x(t)=C_1e^{-\gamma_{(+)}t}+C_2e^{-\gamma_{(-)}t}\to\gamma_{(\pm)}=-\gamma\pm\sqrt{\gamma^2-\omega_0^2}$

$\omega_0=\gamma\to x(t)=\left(C_1+C_2t\right)e^{-\gamma t}$

$\ddot{x}+2\gamma\dot{x}+\omega_0^2 x=B\sin\left(\Omega t+\theta_0\right)$ $\quad x(t)=A_P\sin\left(\Omega t+\theta_0-\varphi_P\right)$

$\tan\varphi_P=\frac{2\gamma\Omega}{\omega_0^2-\Omega^2}$ $\quad Z=\sqrt{4\gamma^2+\left(\Omega-\frac{\omega_0^2}{\Omega}\right)^2}$ $\quad F_0=mB\to A_P=\frac{F_0}{\Omega Z}$

$\Omega\,|\,A_P\max\to\Omega^2=\omega_0^2-2\gamma^2$ $\quad \Omega\,|\,v=\Omega A_P\max\to\Omega=\omega_0$

$\bar{P}=\frac{1}{2}F_0A_p\Omega\sin\varphi_P$

Tema 7. Ones mecàniques

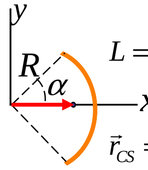
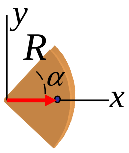
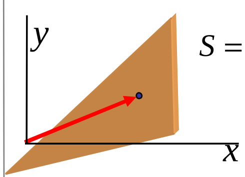
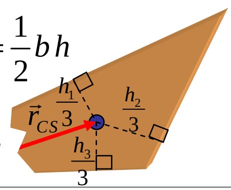
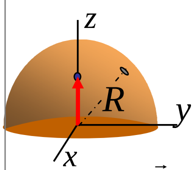
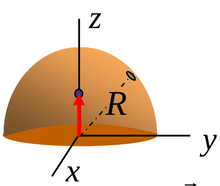
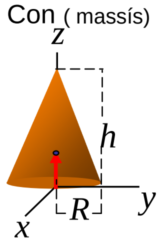
$\frac{\partial^2y}{\partial x^2}-\frac{1}{v^2}\frac{\partial^2y}{\partial t^2}=0$ $\quad y(x,t)=A\sin\left(\omega\left(t-\frac{x}{v}\right)+\varphi_0\right)=A\sin\left(\omega t-kx+\varphi_0\right)$

$\omega=\frac{2\pi}{T}=2\pi f$ $\quad k=\frac{2\pi}{\lambda}$ $\quad v=\frac{\lambda}{T}=f\lambda=\frac{\omega}{k}$ $\quad$ longitudinals: $\Delta\rho=-\rho_0\frac{dy}{dx}$; corda: $v=\sqrt{\frac{T}{\lambda}}$

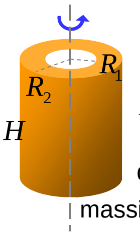
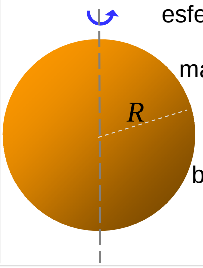
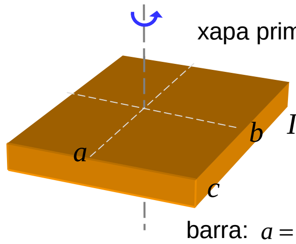
$\omega_R=\omega_T=\omega_I$ $\quad A_R=\frac{v_2-v_1}{v_1+v_2}A_I$ $\quad A_T=\frac{2v_2}{v_1+v_2}A_I$ $\quad I=\frac{dE}{ds\,dt}=\frac{1}{2}\rho\omega^2vA^2$

$y=A_R\sin\alpha_R$ $\quad A_R=\sqrt{A_1^2+A_2^2+2A_1A_2\cos(\alpha_2-\alpha_1)}$ $\quad \tan\alpha_R=\frac{A_1\sin\alpha_1+A_2\sin\alpha_2}{A_1\cos\alpha_1+A_2\cos\alpha_2}$

Centres de massa d'alguns cossos homogènis

Arc de circunferència	Sector de cercle	Triangle arbitrari
 $L=2R\alpha$; $\vec{r}_{CS}=\left(\frac{R\sin\alpha}{\alpha}\ ,\ 0\right)$	 $S=R^2\alpha$; $\vec{r}_{CS}=\left(\frac{2R\sin\alpha}{3\alpha}\ ,\ 0\right)$	 $S=\frac{1}{2}b\,h$ 
Closca semiesfèrica (sense la tapa inferior)  $S=2\pi R^2$; $\vec{r}_{CS}=\left(0,0,\frac{R}{2}\right)$	**Semiesfera** (massís)  $V=\frac{2\pi R^3}{3}$; $\vec{r}_{CS}=\left(0,0,\frac{3R}{8}\right)$	**Con** (massís)  $V=\frac{\pi R^2h}{3}$; $\vec{r}_{CS}=\left(0,0,\frac{h}{4}\right)$

Moments d'inercia respecte d'un eix d'alguns cossos homogènis

cilindre gruixut	esfera	xapa prima/gruixuda
 $I=\frac{1}{2}m\left(R_1^2+R_2^2\right)$; disc: $H=0$; massis: $R_1=0$ prim: $R_1=R_2$	 massisa $I=\frac{2}{5}mR^2$; buida $I=\frac{2}{3}mR^2$	 $I=\frac{1}{12}m\left(a^2+b^2\right)$; barra: $a=0$; $c=0$