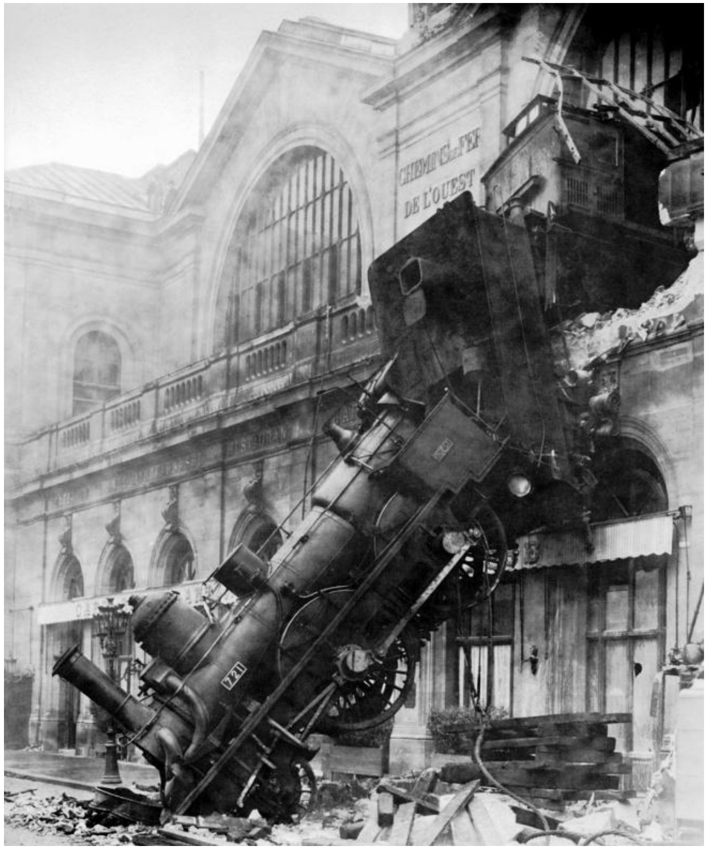
Unit 1



Introduction

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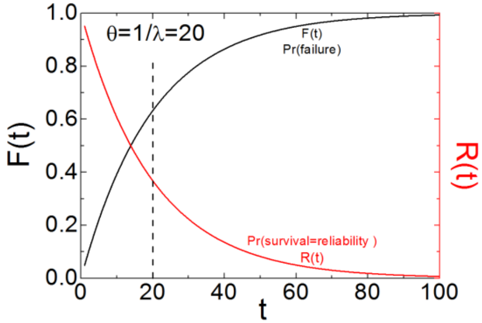
Summary

0.- Remember
1.- Bayes theorem
2.- Maximum likelihood method
3.- Estimation of reliability parameters from tests
4.- Confidence limits of parameters
5.- Accelerated life testing
6.- Determination of distribution models
7.- Empirical determination of survivor function
8.- Reliability growth
9.- Strength-stress models

What you have done

You know Reliability or Failure

$$R(t), F(t)$$



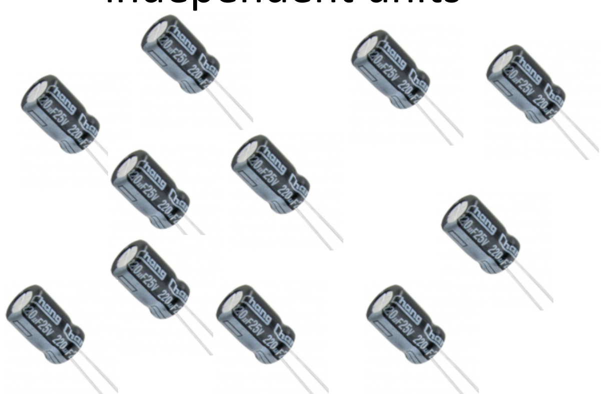
You apply to one component

or to a system (series/parallel)

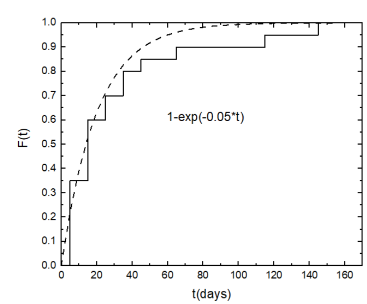
frequentist approach $P(a) = \frac{cases\_observed}{total\_cases}$ will joint the two worlds

What we are going to do

You have a lot of identical and independent units

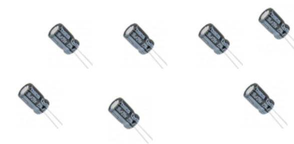


You want to obtain R(t), F(t) from experiment

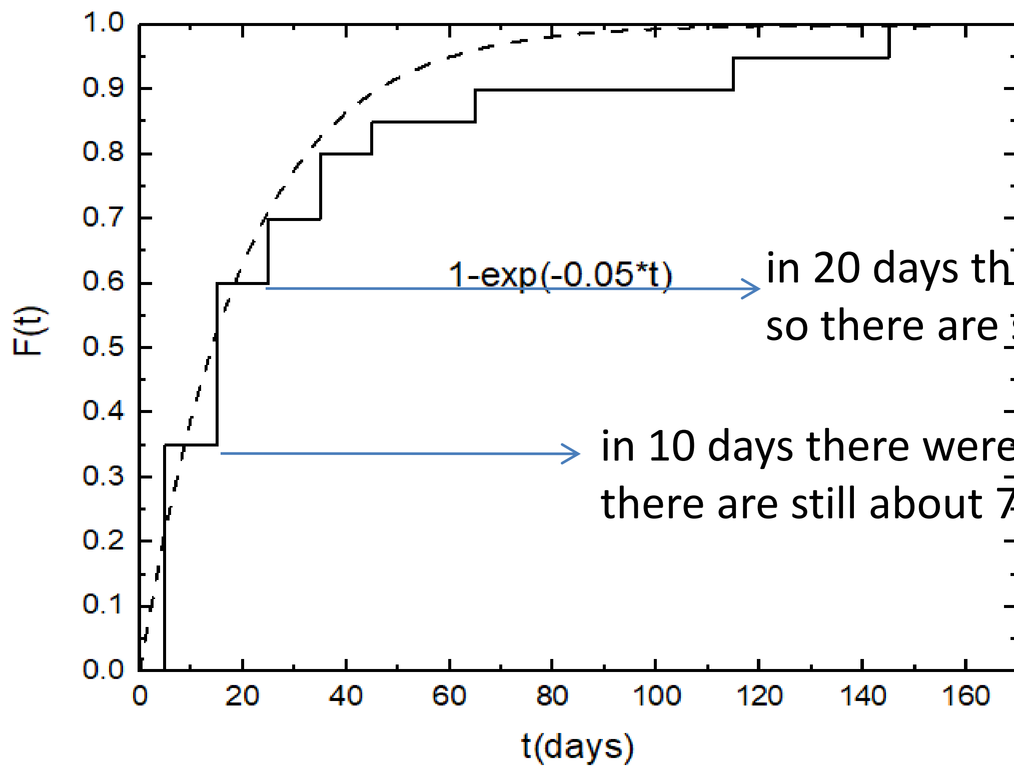


and then find a model for a single unit

... Imagine that you have n different and independent units
Let’s assume, for simplicity, that you have n=100 components



n·F(t): total number of components that failed as a function of time
n·R(t): total number of components still working at a given time

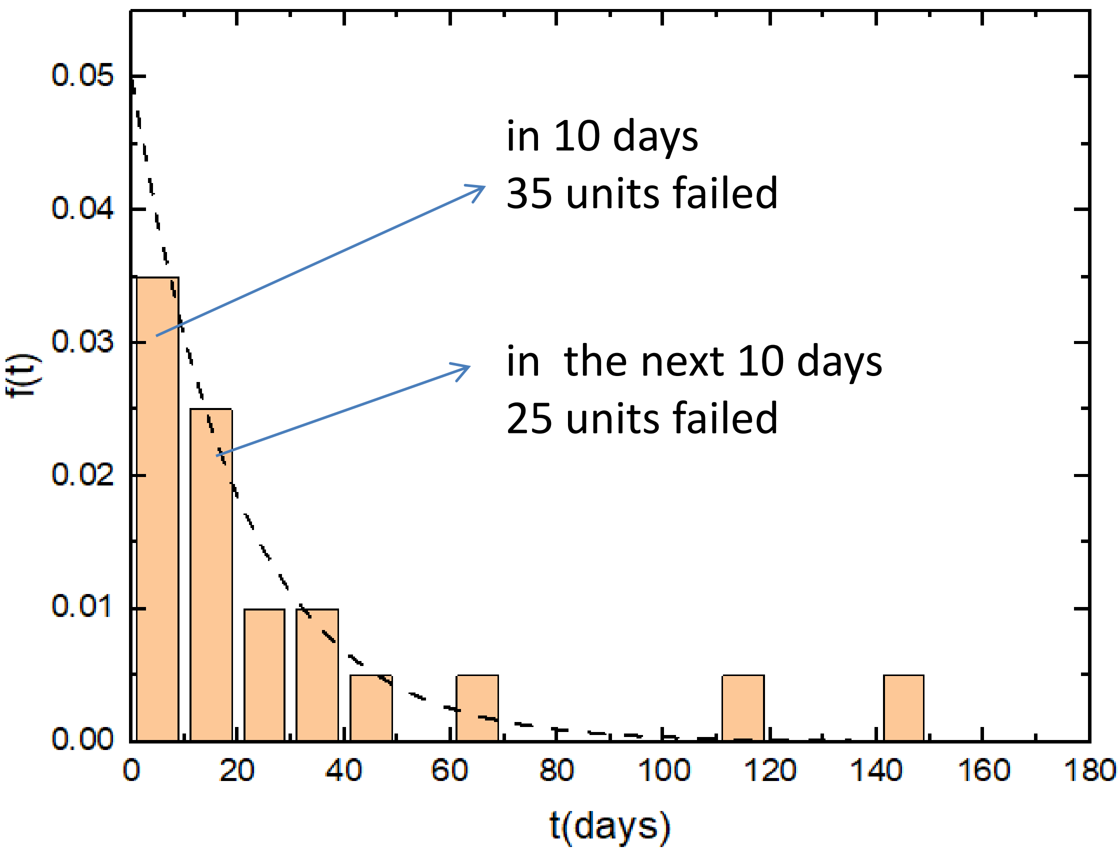


in 20 days there were about 60 failures, so there are still about 40 units working

in 10 days there were about 35 failures, so there are still about 70 units working

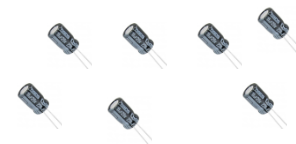
... Imagine that you have n different and independent units
Let's assume, for simplicity, that you have n=100 components

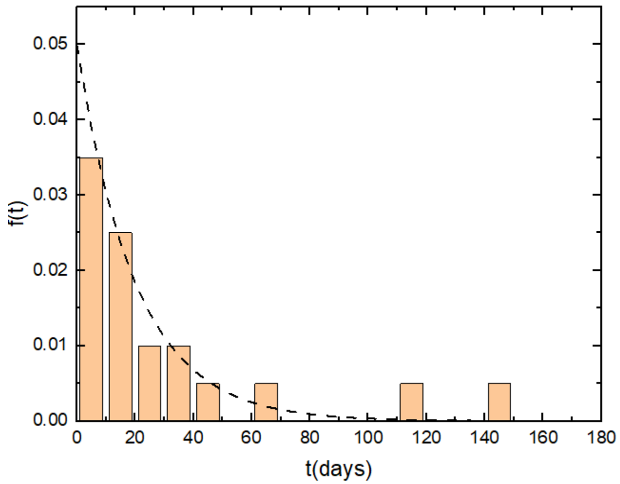
N·f(t): number of failures at each time interval

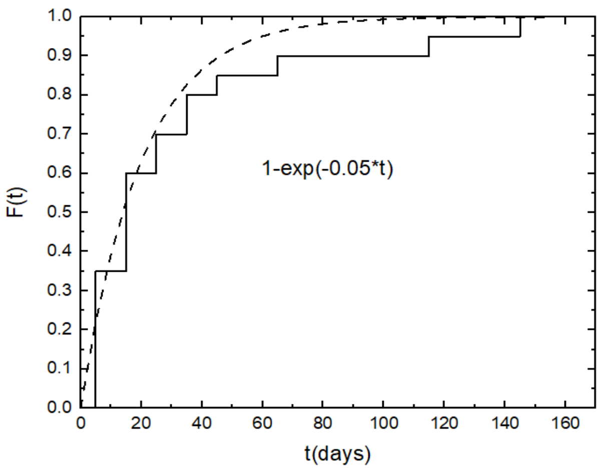


of course 35+25=60 units failed in total: this is the integral!!!!

... Imagine that you have n different and independent units
Let's assume, for simplicity, that you have n=100 components







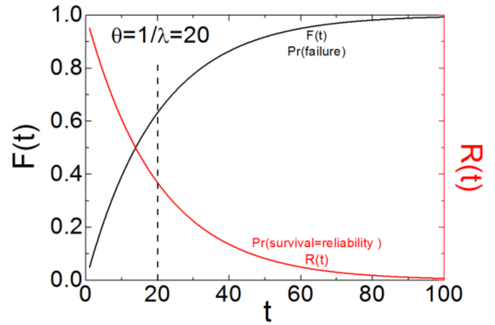
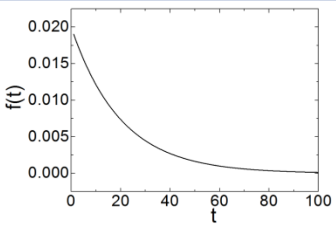
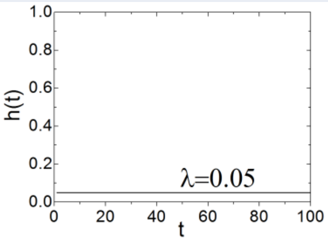
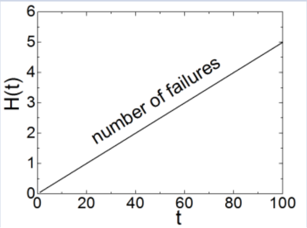
N·f(t)
number of failures at each time interval

$$h(t)=\frac{f_T(t)}{R(t)}=\frac{N\cdot f_T(t)}{N\cdot R(t)}$$

N·R(t)
total number of components that failed as a function of time

therefore h(t) is the proportion of components that failed

Summary of important quantities

	Description	example	graph
cdf $R_T(t)$	Reliability function or Survival function	$R_T(t) = e^{-\lambda t}$	
cdf $F_T(t)$	Failure probability Lifetime distribution function	$F_T(t) = 1 - e^{-\lambda t}$	
pdf $f_T(t)$	Probability to fail Between t and t+dt if it didn't fail up to now	$f_T(t) = \lambda e^{-\lambda t}$	
Pdf $\lambda(t)$ $h(t)$	Failure rate or Hazard function	$h(t) = \lambda$	
cdf $H(t)$	Cumulative hazard function	$H(t) = \lambda \cdot t$	

Summary of important relations

$$R_T(t) = 1 - F_T(t)$$

Survival function /
Faulure probability

$$f_T(t) = \frac{dF_T(t)}{dt}$$

$$F_T(t) = \int_0^t f_T(t)\,dt$$

$$f_T(t)$$

failure probability

$$h(t) = \frac{f_T(t)}{R(t)}$$

cumulative hazard function

$$H(t)$$

$$h(t) = \frac{dH(t)}{dt}$$

$$H(t) = \int_0^t h(t)\,dt$$

failure ratio

$$h(t)$$

Summary of important relations

$$F_T(t) = 1 - e^{-\lambda t}$$

$$f_T(t) = \frac{dF_T(t)}{dt}$$

$$f_T(t)$$

$$F_T(t) = \int_0^t f_T(t)\,dt$$

Failure probability

Failure probability at t

$$h(t) = \frac{f_T(t)}{R(t)}$$

failure ratio

$$h(t) = \frac{dH(t)}{dt}$$

$$H(t) = \lambda \cdot t$$

$$h(t) = \lambda = \frac{failures}{time}$$

$$H(t) = \int_0^t h(t)\,dt$$

$$MTBF = \frac{1}{\lambda} = \frac{time}{failures}$$

Mean **T**ime **B**etween **F**ailures

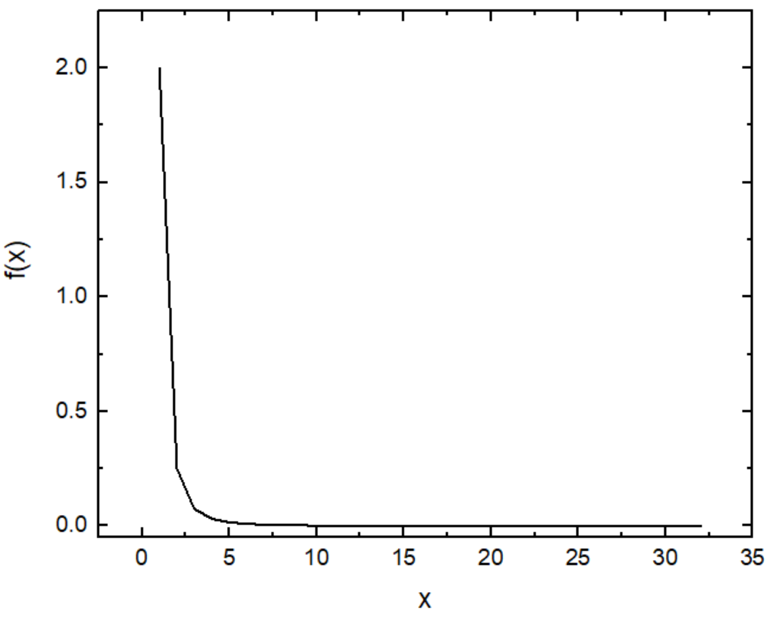
LINEARIZATION

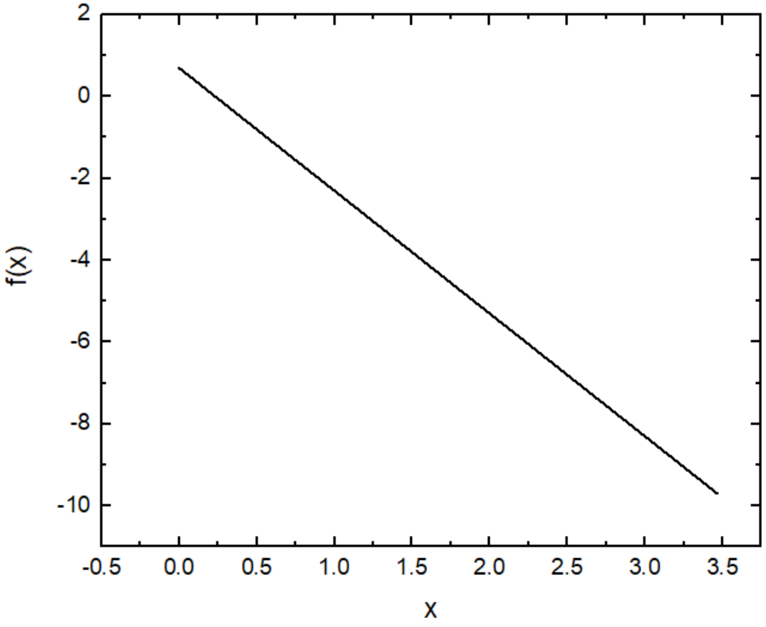
An exponential function is not easy to visualize

One “trick” is to linearize the function so that it is a stright line and parameters are much easier to obtain

$$f(x) = Ax^{-B} \Rightarrow \ln(f(x)) = \ln\left(Ax^{-B}\right) = \ln(A) - B\ln(x)$$

log-log scale





LINEARIZATION

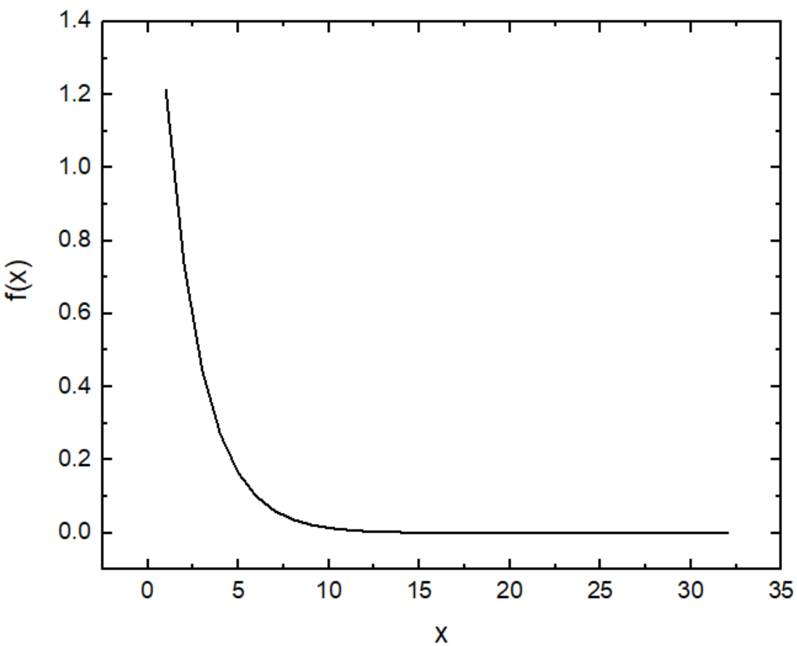
An exponential function is not easy to visualize

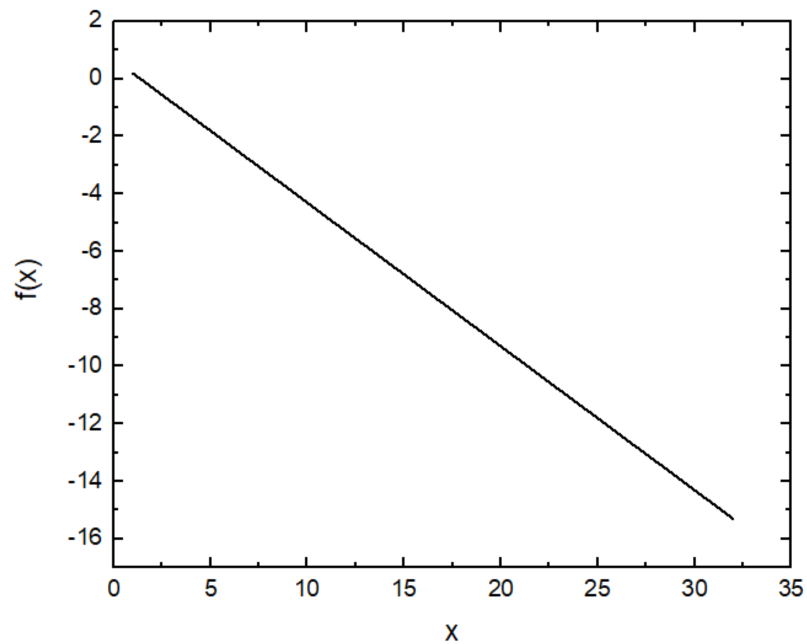
One “trick” is to linearize the function so that it is a stright line and parameters are much easier to obtain

$$f(x) = Ae^{-xB}$$

$$\ln(f(x)) = \ln\left(Ae^{-xB}\right) = \ln(A) - B\cdot x$$

semi log scale





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