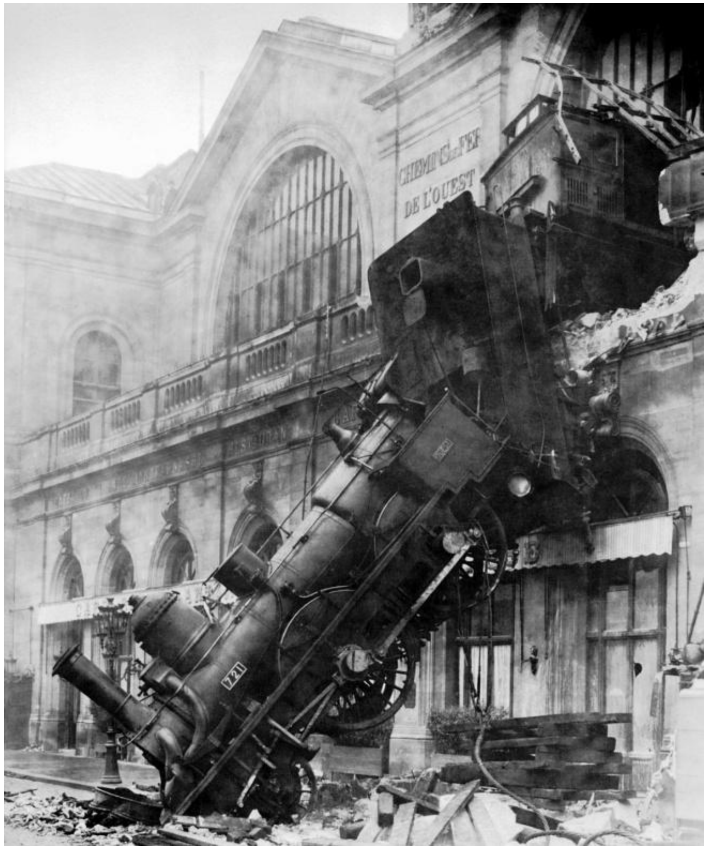
Unit 5



Empiric survival function

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Summary

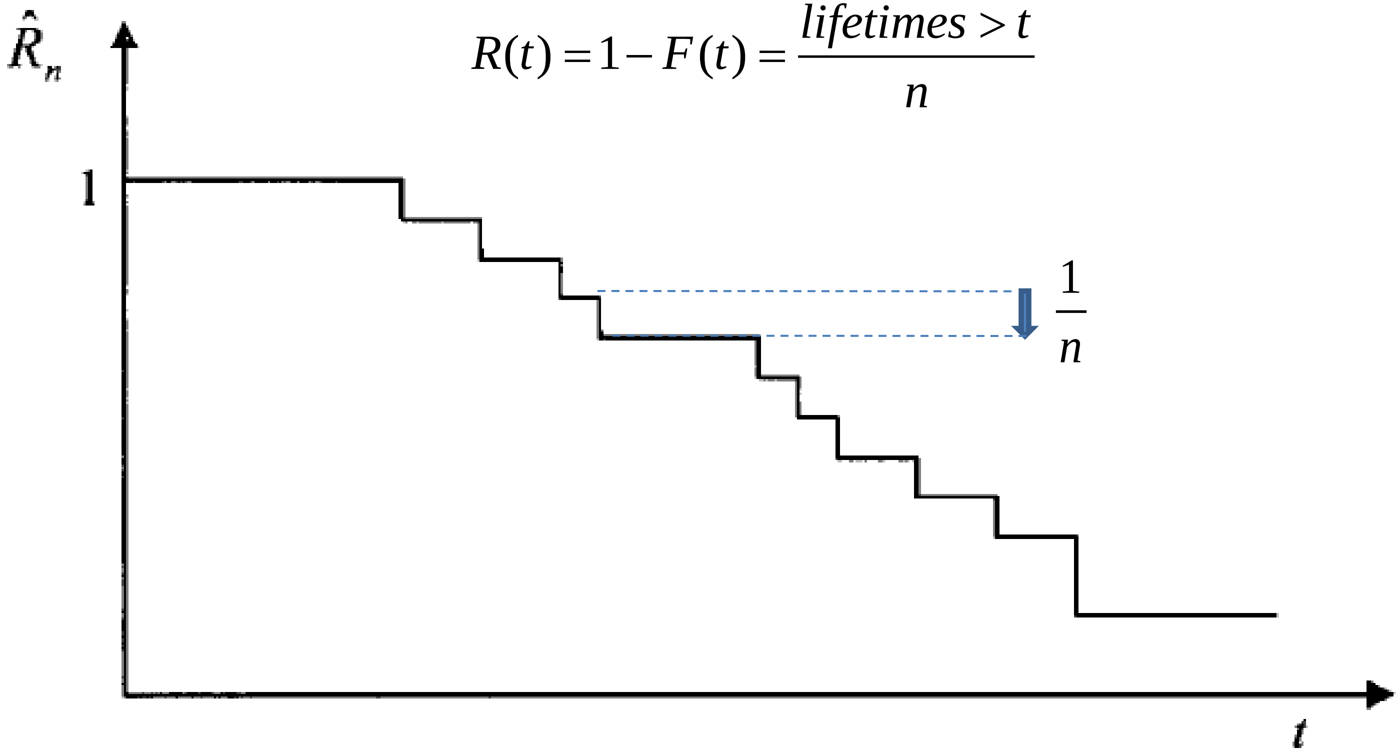
1.- Bayes theorem
2.- Maximum likelihood method
3.- Estimation of reliability parameters from tests
4.- Confidence limits of parameters
5.- Accelerated life testing
6.- Determination of distribution models
7.- Empirical determination of survival function
8.- Reliability growth
9.- Strength-stress models

ok... but i don't want to find a model. I want to directly the experimental CDF...

For a non-censored experiment this is quite easy!

Let's do it with the survival function R(t):
t_i : is the lifetime of the unit i=1,2,3,... n

$$R(t) = 1 - F(t) = \frac{lifetimes > t}{n}$$



It must go down from 1 to 0 in n steps, therefore each step MUST be of height 1/n

KAPLAN-MEIER estimator

ok... but i don't want to find a model. I want to directly the experimental CDF...

For a censored experiment is not that easy!

Let's do it with the survival function R(t):

t_i : is the lifetime of the unit i=1,2,3,... n

$(u_j, u_{j+1}]$: is an interval between u_j and u_{j+1} small enough that only a t_i falls into each interval

Let's calculate R(t) at a time t_m

$$R(t_m) = P(T > t_m) = P(T > u_1 \mid T > u_0) \cdot P(T > u_2 \mid T > u_2) \cdot \ldots P(T > t_m \mid T > u_m)$$

$$R(t_m) = P(T > t_m) = \prod_{i=0}^{m} P_i$$

$$P_i = P(T > u_{i+1} \mid T > u_i)$$

KAPLAN-MEIER estimator

The goal is to calculate P_i

$$R(t_m) = P(T > t_m) = \prod_{i=0}^{m} P_i$$

$$P_i = P(T > u_{i+1} \mid T > u_i)$$

remember:

Intervals $(u_j, u_{j+1}]$ are small enough so that maximum 1 failure occurs

1. If neither failure or censoring occurs $P_i=1$

2. If censoring occurs, no recording of failures occur, and again $P_i=1$

3. Imagine that a failure occurs in $(u_j, u_{j+1}]$.
   The number of units at risk before are n_j
   The number of units at risk after are n_j-1

$$P_i = P(T > u_{i+1} \mid T > u_i) = \frac{n_j - 1}{n_n}$$

Therefore the intervals where no failure occur can be misregarded

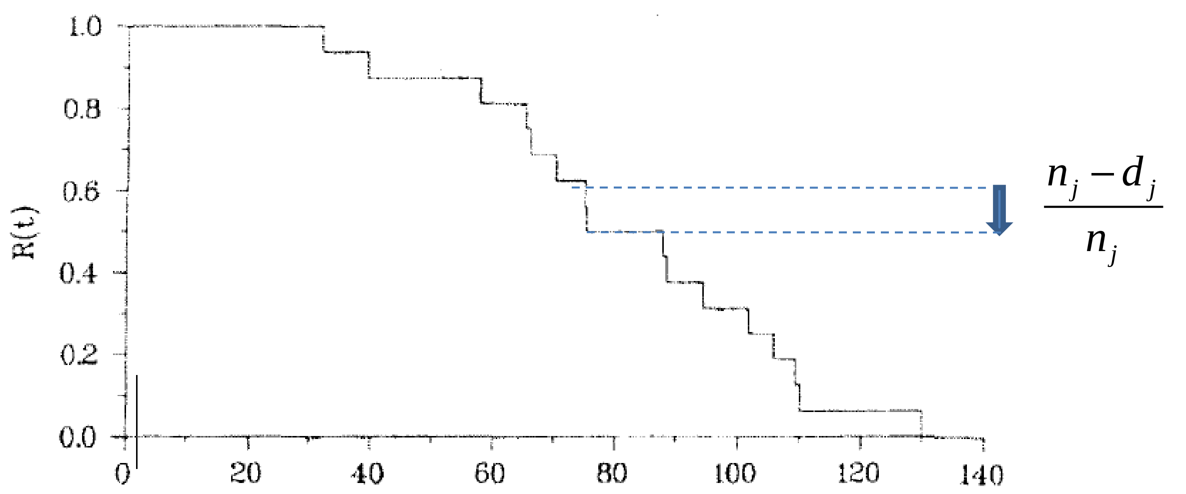
KAPLAN-MEIER estimator

Therefore, R(t) will go down each time a component fail, and R(t) can be represented as:

$$R\left(t_m\right) = P\left(T > t_m\right) = \prod_{i=0}^{m} P_i = \prod_{j=1}^{n_f} \frac{n_j - 1}{n_j}$$

If more than on unit fails at interval i, then if d_j units fail:

$$R\left(t_m\right) = P\left(T > t_m\right) = \prod_{i=0}^{m} P_i = \prod_{j=1}^{n_f} \frac{n_j - d_j}{n_j}$$



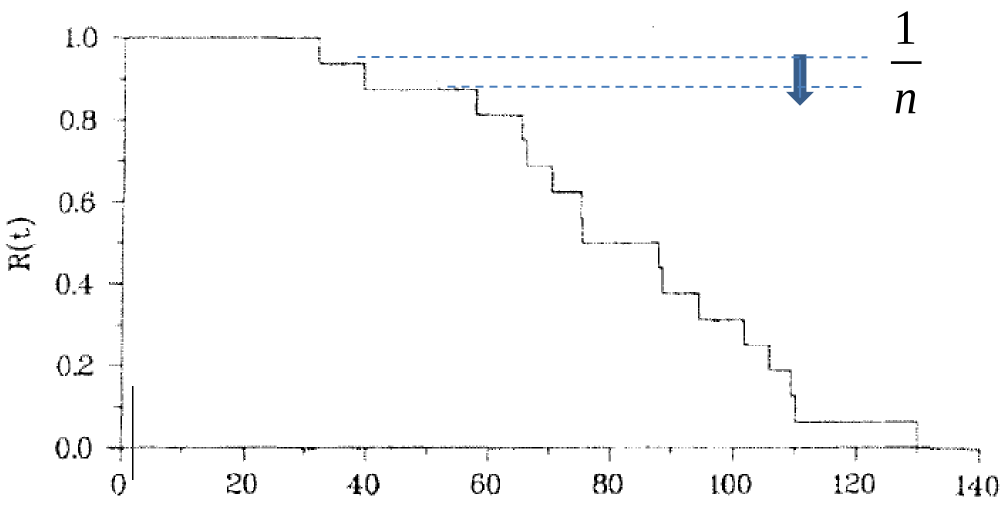
KAPLAN-MEIER estimator

EXAMPLE:

A test is carried out for n=16 units, obtaining the folloring failure times

31.7 ; 39.2 ; 57.5 ; 65.0 ; 65.8 ; 70.0 ; 75.0 ; 75.2 ;
87.5 ; 88.3 ; 94.2 ; 101.7 ; 105.8 ; 109.2 ; 110.0 ; 130.0

Calculate the survival function R(t)



KAPLAN-MEIER estimator

EXAMPLE (same times as before):
A test is carried out for n=16 units, obtaining the folloring failure times

31.7 ; 39.2 ; 57.5 ; 65.8 ; 70.0 ; 105.8 ; 110.0

The rest are censored tests Calculate the survival function R(t)

Rank j	Inverse Rank n-j+1	Ordered Failure and Censoring Times t_j	$\hat{p}_j$	$\hat{R}(t_{(j)})$
0	-	-	1	1.000
1	16	31.7	15/16	0.938
2	15	39.2	14/15	0.875
3	14	57.2	13/14	0.813
4	13	65.0*	1	0.813
5	12	65.8	11/12	0.745
6	11	70.0	10/11	0.677
7	10	75.0*	1	0.677
8	9	75.2*	1	0.677
9	8	87.5*	1	0.677
10	7	88.3*	1	0.677
11	6	94.2*	1	0.677
12	5	101.7*	1	0.677
13	4	105.8	¾	0.508
14	3	109.2*	1	0.508
15	2	110.0	½	0.254
16	1	130.0*	1	0.254

t	$\hat{R}(t)$
$0 \le t < 31.7$	=1
$31.7 \le t < 39.2$	15/16=0.938
$39.2 \le t < 57.5$	15/16·14/15=0.875
$57.5 \le t < 65.8$	15/16·14/15·13/14=0.813
$65.8 \le t < 70.0$	15/16·14/15·13/14·11/12=0.745
$70.0 \le t < 105.8$	15/16·14/15·13/14·11/12·10/11=0.677
$105.8 \le t < 110.0$	15/16·14/15·13/14·11/12·10/11·3/4=0.508
$110.0 \le t$	15/16·14/15·13/14·11/12·10/11·3/4·1/2=0.254

KAPLAN-MEIER estimator

EXAMPLE (same times as before):
A test is carried out for n=16 units, obtaining the folloring failure times

31.7 ; 39.2 ; 57.5 ; 65.8 ; 70.0 ; 105.8 ; 110.0

The rest are censored tests   Calculate the survival function R(t)