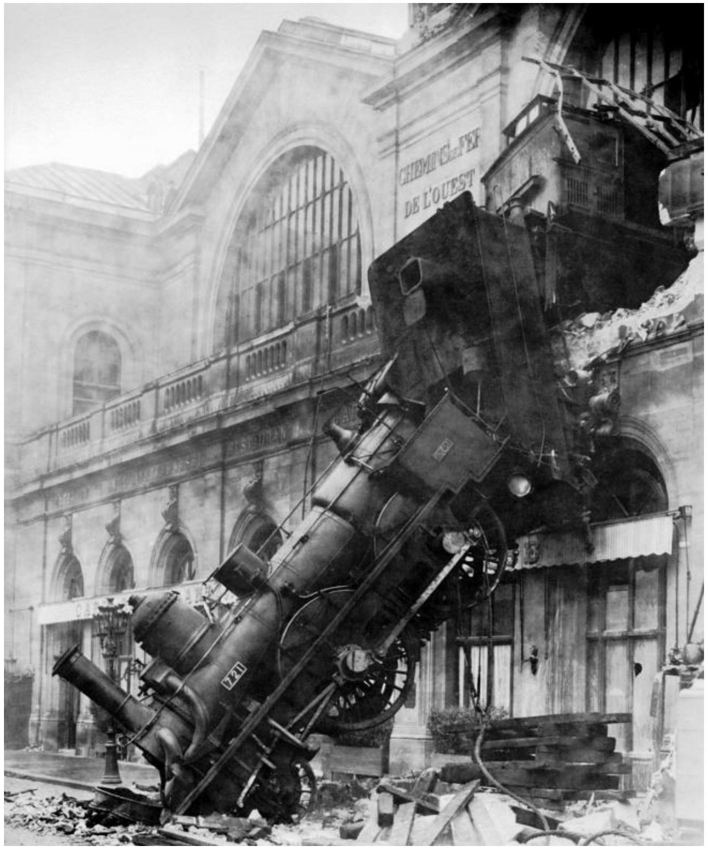
Unit 6



Reliability growth

Luis Carlos Pardo

Escola d'Enginyeria de Barcelona Est

Summary

1.- Bayes theorem
2.- Maximum likelihood method
3.- Estimation of reliability parameters from tests
4.- Confidence limits of parameters
5.- Accelerated life testing
6.- Determination of distribution models
7.- Empirical determination of survivor function
8.- Reliability growth
9.- Strength-stress models

Reliability growth:

Engineering changes cause an increase of reliability

Can be quantified:

- Cumulative number of failures as a function of time
- Failure rate **h(t)=λ(t)** as a function of time
- Mean time between failures (**MTBF**) as a function of time

We need, in any case, a reliability growth model

We do it using the MTBF, remember that:

$$MTBF_{cumulative} = \frac{Total\_operating\_period}{Number\_of\_failures} = \frac{t}{H(t)}$$

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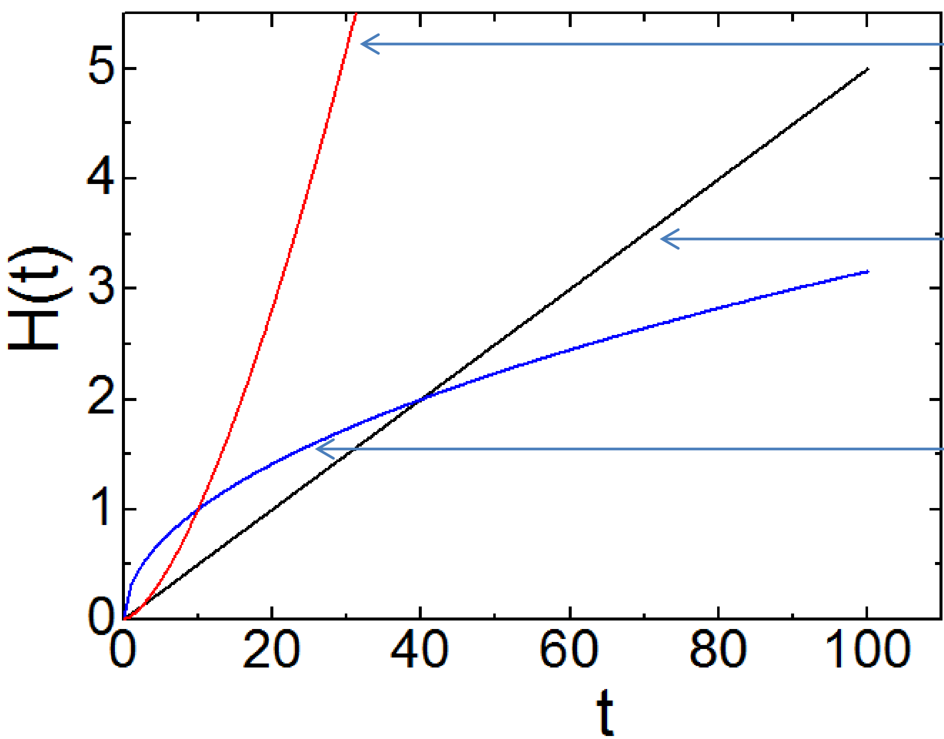


Duane model (β>0)

$$H(t) = \left(\frac{t}{\tau}\right)^{\beta}$$

Exponential (β=0)

$$H(t) = \lambda \cdot t = \frac{t}{\tau}$$

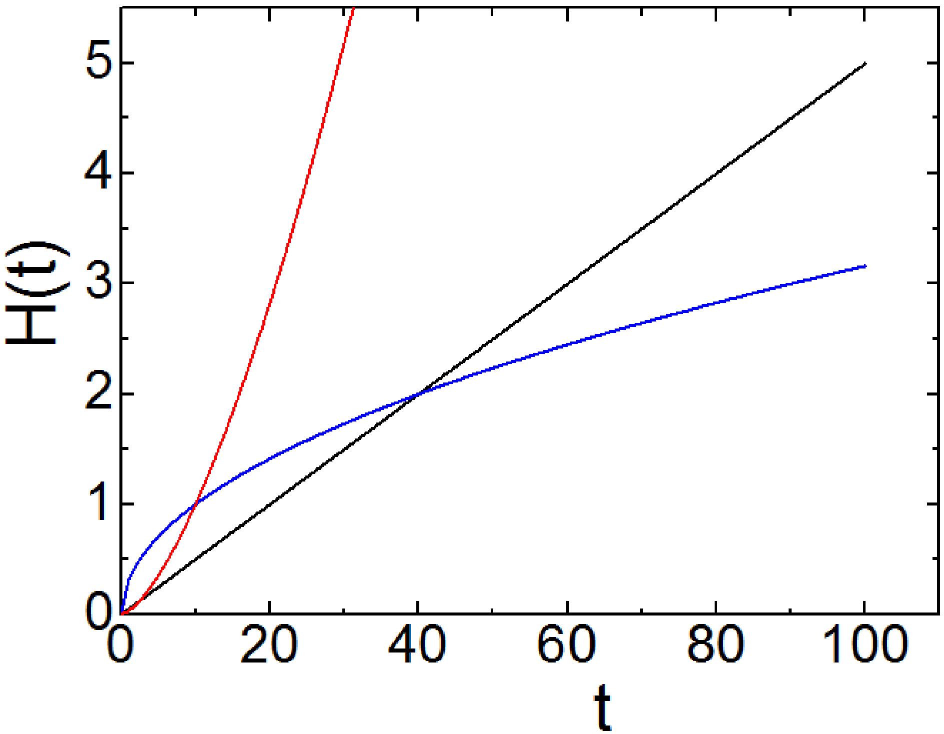
Duane model (β<0)

$$H(t) = \left(\frac{t}{\tau}\right)^{\beta}$$

Let's find the parameters of the Duane model: **Maximum likelihood**

Duane model (β>0)

$$H(t) = \left(\frac{t}{\tau}\right)^{\beta}$$



Duane model (β<0)

$$H(t) = \left(\frac{t}{\tau}\right)^{\beta}$$

Calculating de likelihood, making the logariothm and minimizing:

$$\hat{\beta} = \frac{n}{\sum_{i=1}^{n} \ln\left(\frac{T}{t_i}\right)} \qquad \hat{\alpha} = \frac{T}{n^{1/\beta}}$$

Let's find the parameters of the Duane model: **Least squares**

Example:
Let's calculate the parameters of the Duane analysis for the following data

(1)	(2)	(3)	(4)	(5)	(6)
Month of Operation	Hours of Operation	Cumulative Hours t	No. of failures	Cumulative Number of failures $H(t)$	Cumulative MTBF $t/H'(t)$
1	541	541	3	3	180.3
2	1171	1712	5	8	214.0
3	1939	3651	4	12	304.3
4	2403	6054	1	13	465.7
5	1718	7772	2	15	518.1
6	2206	9978	2	17	586.9
7	1366	11244	3	20	562.2
8	1529	12873	0	20	643.7
9	1449	14322	2	22	651.0
10	1451	15773	2	24	657.2

Let’s find the parameters of the Duane model: **Least squares**

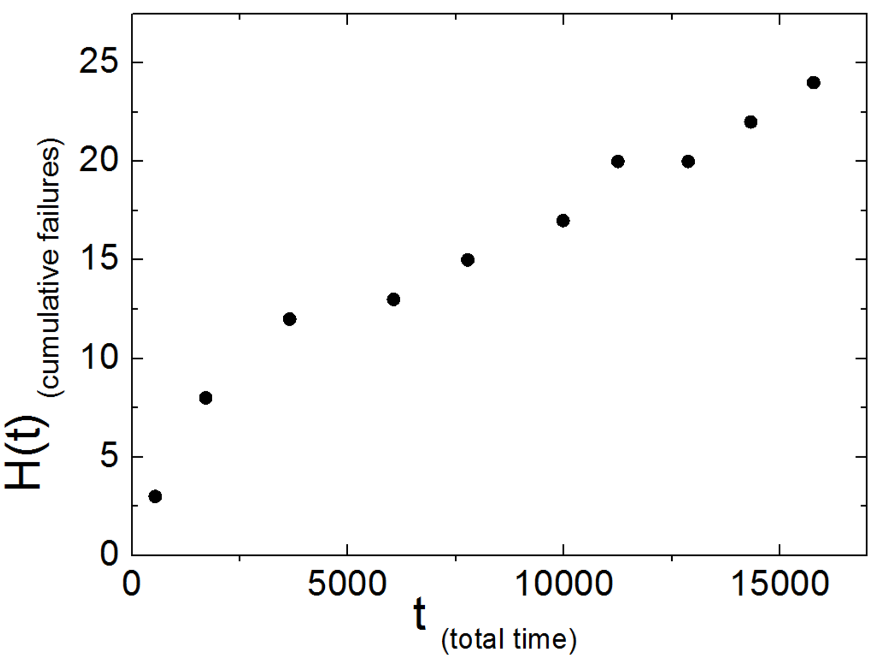
Let’s first do the figure:

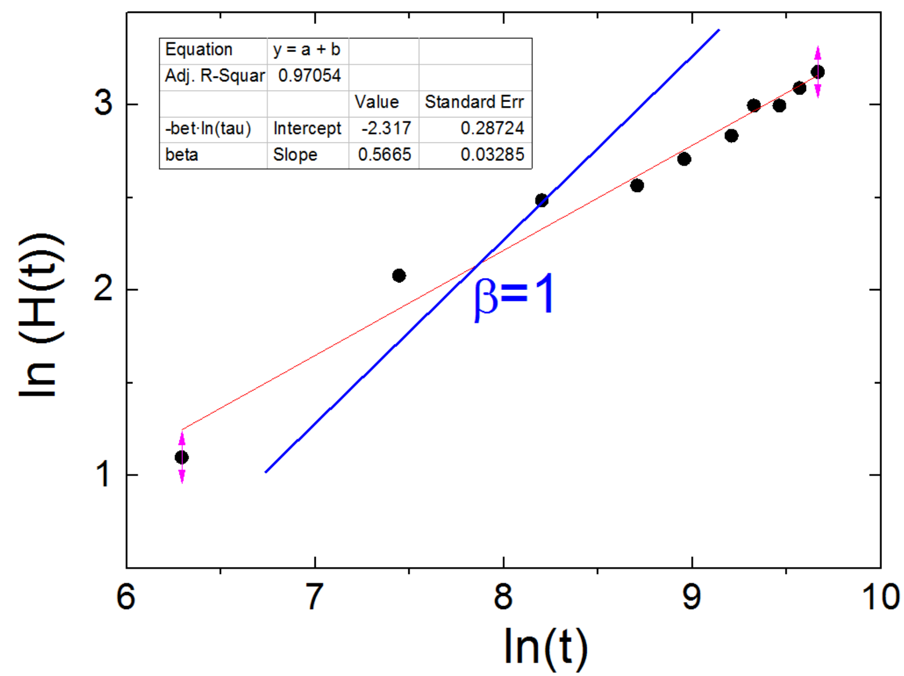
Duane Model

$$H(t) = \left(\frac{t}{\tau}\right)^{\beta}$$

linearized Duane Model

$$\ln\left(H(t)\right) = \beta \ln t - \beta \ln \tau$$





β<1 : reliability growth

Let's now add MONEY

Imagine a system whose failure causes production stoppage:

- Failure costs *Cr* (include production stoppage)
- Complete overhaul costs *Co* (includes lost of production, replacement and labour)

What is the optimum overhaul policy?

The total cost is

$$C(t) = C_r H(t) + C_o$$

C_o ← cost total overhaul

$H(t)$ ↑ # units that fail

C_r ↑ cost to repair

And we just showed the Duane model for H(t)

$$H(t) = \left(\frac{t}{\tau}\right)^{\beta}$$

Therefore , the cost/unit operating time is

$$\gamma(t) = \frac{C(t)}{t} = \frac{C_r \left(\frac{t}{\tau}\right)^{\beta} + C_o}{t}$$

and we want o minimize this cost...

Therefore , the cost/unit operating time is $$\gamma(t) = \frac{C(t)}{t} = \frac{C_r\left(\frac{t}{\tau}\right)^{\beta} + C_o}{t}$$

and we want o minimize this cost...

We solve the equation $$\frac{d\gamma(t)}{dt} = 0$$

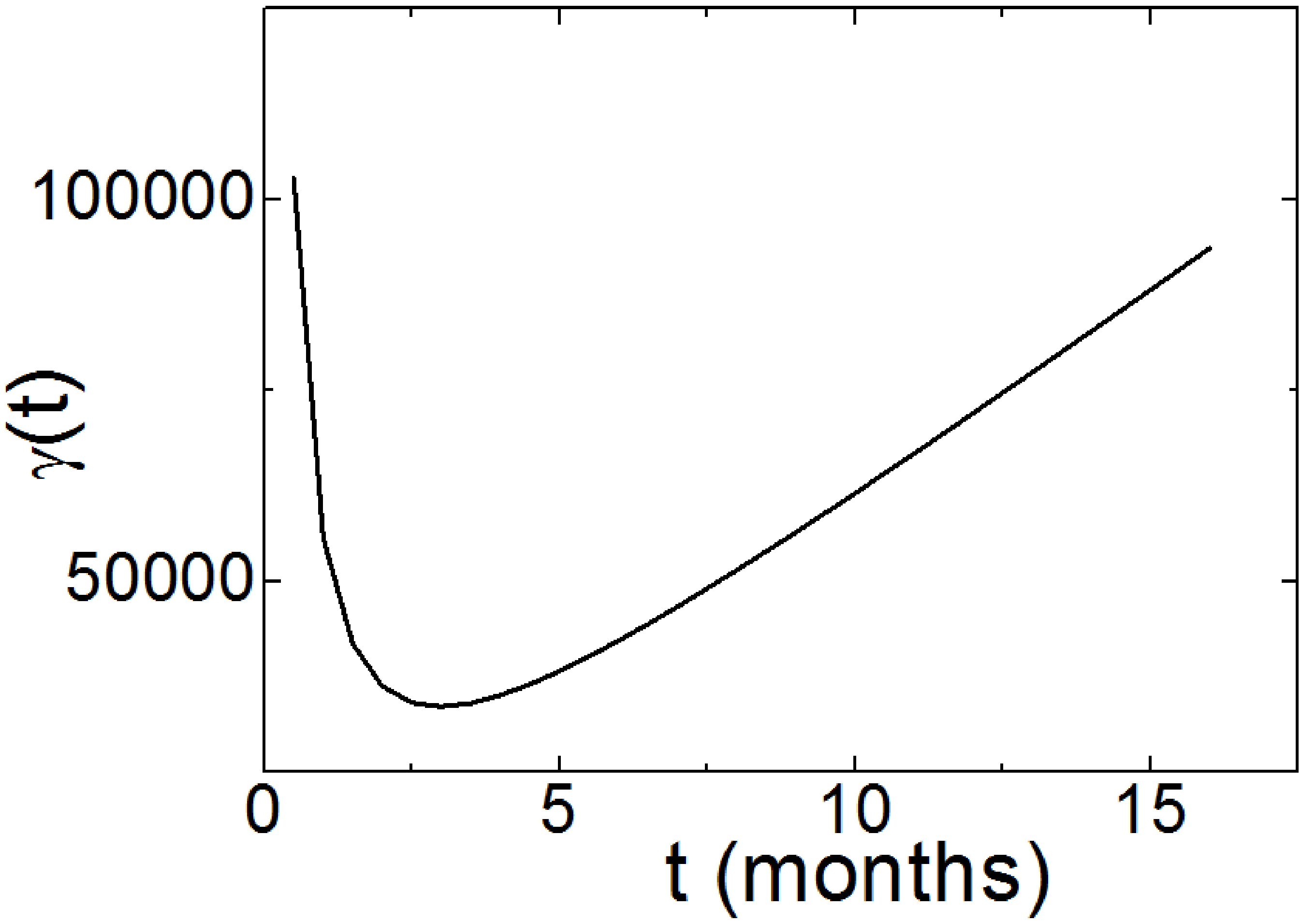
And we obtain that the optimal overhaul time is

$$t^* = \tau\left[\frac{\alpha^{\beta} C_o}{C_r(\beta - 1)}\right]^{1/\beta}$$

The overhaul time increases with Co... since it is expensive to do,
and increases with Cr, since if it is expensive is better to cahnge it

And the time is ∞ for β=1 (exponential)... therefore do never change

We can learn more looking at the cost/unit operating time



... when in doubt, do it later!